



## I Semester B.C.A. Degree Examination, November/December 2016

(CBCS) (F+R)

(2014-15 &amp; Onwards)

## BCA - 105 : DISCRETE MATHEMATICS

Time : 3 Hours

Max. Marks : 100

**Instruction : Answer all Sections.**

## SECTION - A

I. Answer any ten : (10×2=20)

1) If  $A = \{x | x \in \mathbb{N} \text{ and } x < 3\}$  and  $B = \{0, 1, 3\}$ . Find  $A - B$ .2) If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4, 5\}$  and  $C = \{0, 2, 3\}$ , find  $(A \cap B) \times C$ .3) Construct truth table for the proposition  $p \vee \sim q$ .4) Find  $x, y, z$  if  $\begin{bmatrix} 4-y & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} -1 & z+1 \\ 1 & 5 \end{bmatrix}$ .5) If  $A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 0 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ , find  $AB$ .6) Find the characteristic equation of the matrix  $\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ .7) Prove that  $\log_b a \cdot \log_c b \cdot \log_a c = 1$ .8) Find  $n$  if  $2 \cdot {}^n P_3 = {}^n P_5$ .9) On the set of integers  $Z$ , the binary operation  $*$  is defined by

$$a * b = \frac{ab}{3}, \forall a, b \in Z. \text{ Find identity element.}$$

10) If  $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$  find unit vector along  $\vec{a} - \vec{b}$ .

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11) Find the midpoint of line joining  $(-2, 8)$  and  $(1, -2)$ .

12) Find the equation of the line passing through  $(-1, 2)$  and having slope 3.

## SECTION - B

II. Answer any six of the following :

(6×5 =30)

13) If  $A = \{1, 4\}$ ,  $B = \{2, 3, 6\}$ ,  $C = \{2, 3, 7\}$  then verify that  $A \times (B - C) = (A \times B) - (A \times C)$ .

14) Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 4x + 3$  is invertible. Find the inverse of  $f$ .

15) Show that  $p \vee (q \wedge r) \leftrightarrow [(p \vee q) \wedge (p \vee r)]$  is a tautology.

16) If  $(p \rightarrow q) \wedge (p \wedge r)$  is given to be false, find the truth values of  $p, q, r$ .

17) Write the truth table of  $(p \vee q) \vee \sim p$ . Show that the compound propositions  $p \wedge q$  and  $\sim (p \rightarrow \sim q)$  are logically equivalent.

18) Find the inverse of the matrix  $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ .

19) Using Cramer's rule solve  $3x - y + 2z = 13$ ;  $2x + y - z = 3$ ;  $x + 3y - 5z = -8$ .

20) Verify Cayley Hamilton theorem for the matrix  $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$ .

## SECTION - C

III. Answer any six of the following.

(6×5 =30)

21) If  $\log \left( \frac{a-b}{5} \right) = \frac{1}{2}(\log a + \log b)$ , show that  $a^2 + b^2 = 27 ab$ .

22) Find the number of three digit even numbers that can be formed using 2, 3, 4, 5, 6 repetitions being not allowed.

23) If  ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$  find  $n$ .



- 24) Prove that the set  $G = \{ 3n \mid n \in \mathbb{Z} \}$  is an abelian group w.r.t. addition.
- 25) Prove that the set  $G = \{ 2, 4, 6, 8 \}$  is an abelian group w.r.t. multiplication modulo 10.
- 26) If  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ ,  $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$  find  $(\vec{a} + 2\vec{b}) \cdot (2\vec{a} - \vec{b})$ .
- 27) Show that the points A( 1,2,3), B (2, 3, 1) and C (3,1,2) are vertices of an equilateral triangle.
- 28) If the vectors  $4\hat{i} + 11\hat{j} + m\hat{k}$ ,  $7\hat{i} + 2\hat{j} + 6\hat{k}$  and  $\hat{i} + 5\hat{j} + 4\hat{k}$  are coplanar, then find 'm'.

SECTION - D

IV. Answer **any four** of the following.

(4x5 =20)

- 29) Prove that the points (6, 4), (7, - 2), (5, 1), (4, 7) form vertices of a parallelogram.
- 30) The three vertices of a parallelogram taken in order are (8,5), (-7, -5) and (- 5, 5). Find the co-ordinate of the fourth vertex.
- 31) Find the equation of the locus of a point which moves such that its distance from X-axis is twice its distance from Y-axis.
- 32) Derive the equation of the straight line whose x -intercept is 'a' and y-intercept is 'b'.
- 33) Find 'K' for which the lines  $2x - ky + 1 = 0$  and  $x + (k+1) y - 1 = 0$  are perpendicular.
- 34) Find the equation of straight line which is passing through intersection of the lines  $2x - 3y - 4 = 0$  and  $2x + 2y - 1 = 0$  and perpendicular to the line  $x + 4y - 8 = 0$ .